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ORBITAL CHANGES OF THE GASEOUS RING AROUND B_e STARS

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ABSTRACT

In order to understand the seemingly erratic V/R variations and radial velocity curves of emission edges and central absorption of some Be stars, we have advanced, for the changes of structure of the gaseous ring around these stars, a theory that is based on the interaction between the existing ring and the newly ejected matter from the star. It shows that the structural change of the ring is completely controlled by the angular-momentum input factor and the dissipation factor. In light of this understanding, we have gone on to interpret the observed results of β^1 Mon and π Aqr.

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I. INTRODUCTION

Be stars differ from B stars mainly by the presence of emission lines which are generally agreed to come from the rotating gaseous ring around the central star (Struve 1931). Therefore to understand Be stars it is important to know the formation and evolution of these rings. In previous papers (Huang 1937b, 1975) a scenario has been proposed to show how exactly the ring is formed as a combined result of rapid rotation of the central star and the radiation pressure that exerts on the matter in the equatorial region. The rapid rotation provides the angular momentum while the radiation pressure serves the purpose of lifting up the matter from the stellar surface. Unaware of Huang's papers, Massa (1975) has independently advanced some of Huang's arguments.

That the radiation pressure plays a critical role in the formation of the gaseous envelope around Be stars was pointed out by Gerasimovic (1934) a long time ago, but Huang's idea of the existence of multi-substrata in the gaseous medium around the star had gone a step further than Gerasimovic's. Actually it is easy to see the necessity of introducing multi-substrata if one considers the fact that the specific angular momentum (i.e. angular momentum per unit mass) of a particle in Keplerien motion around the star is proportional to the square root of semi-major axis of the orbit. So even if the star rotates at the break-up velocity at its equator, the ejected matter cannot all be converted into a rotating ring because of the insufficiency of angular momentum. It follows that only a part of the uplifted matter from the star can finally emerge to become the ring and the rest after having transferred a part at least of their angular momentum to the particles in the ring must either escape into infinity or fall back to the star. This idea advanced in 1973 was developed recently into a quantitative theory (Huang 1976) by the use of laws of conservation of energy and angular momentum. The theory shows that

if a rotating star ejects matter, ring formation is a consequence consistent with the laws of conservation of energy and angular momentum.

Once formed, the ring is not an isolated system apart from the gravitational attraction of the central star, because the latter is continually, if not continuously, ejecting new mass, which could interfere with the gas already in the ring. Thus, the ring structure will be modified or even destroyed by its interaction with the newly ejected matter, depending upon the degree of violence of ejection. The present paper is to present a quantitative description of the modification of the ring structure by the newly ejected matter. It is then followed by a demonstration of such modifications in two stars--β^1 Mon and π Aqr--which have so far defied satisfactory interpretation based on a ring with constant semi-major axis and constant eccentricity.

II. MODIFICATION OF THE RING STRUCTURE

In what follows we will use dimensionless variables by adopting the break-up rotational velocity at the equator of the star as the unit of velocity and its equatorial radius as the unit of length. Hence the angular momentum per unit mass of the matter moving at the break-up velocity in the equator will also be the unit of angular momentum per unit mass. In this system of units we denote by $\dot{x}_o$ and $\dot{y}_o$ respectively the tangential and the radial component of ejected velocity of the matter at the equator, and by m, k and e respectively the total mass, the semi-major axis and the eccentricity of the gaseous ring around the star. All of the quantities m, k, and e are considered to change with time either abruptly or slowly as the case may be. Finally it may be noted that the ring must have a lateral extent. So k should be considered as an average value.

Before we formulate the relations that govern the modification of the ring structure by the newly ejected matter from the star we must first state that not all ejected matter will have an effect on the ring, because a part of this matter falls back to the star before it reaches the ring. After the

ejected matter has mixed with the gas in the ring, the mass will be further dissipated as some particles will fall back to the star and others escape into infinity. Thus if we let Δm_j be the ejected mass, Δm the increase of the mass in the ring and Δm_d the mass dissipated (i.e. either falling back to the star or escaping into infinity), we hav $\Delta m_j = \Delta m + \Delta m_d$. It may be pointed out that Δm can be positive, zero, or negative, because there is no a priori reason that Δm_j must be greater than, equal to, or less than Δm_d.

We now assume that the ejection of Δm_j by the star affects the ring structure such that its total mass is changed from m to $m+\Delta m$, its semi-major axis from k to $k+\Delta k$ and its eccentricity from e to $e+\Delta e$. Since Δm_j is the cause of the modification we may use m_j as the independent variable. If furthermore the ejection is continuous and at a constant rate, m_j measures also the time if the rate of ejection is known.

In our unit system the specific angular momentum of Δm_j is $\dot{x}_o$. Let us assume the specific angular momentum of the dissipated mass, namely Δm_d, to be h_d. A consideration of angular momentum before and after the star has ejected Δm_j gives

$$\dot{x}_o \Delta m_j + m\left[k(1-e^2)\right]^{1/2} = (m+\Delta m)\left\{(k+\Delta k)\left[1-(e+\Delta e)^2\right]\right\}^{1/2} + h_d \Delta m_d, \tag{1}$$

or

$$\frac{d}{dm_j}\left\{m\left[k(1-e^2)\right]^{1/2}\right\} = A_1, \tag{2}$$

when it is expressed in a differential form, where we have set

$$A_1 \equiv \dot{x}_o - h_d \frac{dm_d}{dm_j} \tag{3}$$

which represents the angular momentum input to the ring per unit ejected mass.

Similarly a consideration of the dynamical energy before and after the ejection of Δm_j by the star gives

$$\left[\frac{1}{2}(\dot{x}_0^2+\dot{y}_0^2)-1\right]\Delta m_j - \frac{m}{2k} = -\frac{m+\Delta m}{2(k+\Delta k)} + b\,\Delta m_j\,, \tag{4}$$

where all terms except the last one in the right hand side are self-evident. This last term includes the dynamical energies (1) dissipated into heat, (2) carried away by the falling and escaping mass Δm_d, and (3) obtained by the work done by the heat energy of the ejected matter in the process. Under the usual condition b is a positive number, because in the process of mixing of the ejected streams with the gaseous ring the dissipation and the escape of dynamical energy must be more effective than the conversion of heat into dynamical energy. We may again write equation (4) in a differential form as

$$\frac{d}{dm_j}\left(\frac{m}{k}\right) = 2C_1\,, \tag{5}$$

where

$$C_1 = 1-\frac{1}{2}(\dot{x}_0^2+\dot{y}_0^2)+b\,, \tag{6}$$

which represents the dynamical energy that the ring dissipates for each unit of ejected mass. Thus we see that the modification of the ring structure by the newly ejected matter is controlled by two parameters, the angular momentum input factor A_1 and the energy dissipation factor C_1.

If we denote by m_o, k_o and e_o the initial values of m, k and e respectively of the ring before the ejection has started, i.e. when $m_j = 0$, integration of equations (2) and (5) yields

$$m\left[k(1-e^2)\right]^{1/2} = m_0 E_0 + A m_j \,, \tag{7}$$

and

$$\frac{m}{k} = \frac{m_0}{k_0} + 2C m_j \,. \tag{8}$$

where

$$E_0 = \left[k_0(1-e_0^2)\right]^{1/2}. \tag{9}$$

Here A and C are respectively values of A_1 and C_1 averaged over the entire process in which a total mass of m_j has been ejected from the star.

In general we may write

$$m = m_0 + \gamma m_j \,, \tag{10}$$

where γ is the fraction of ejected mass that enters into the ring. Hence γ is at most equal to 1 but can be negative if the ejected matter induces the ring to dissipate more mass than is acquired. If we further write

$$\beta = \frac{m_j}{m_0} \tag{11}$$

k and e of equations (7) and (8) may be expressed in terms of β which replaces m_j as the independent variable in the following discussion.

In order to show the effect of ejected mass on the ring structure most comprehensively we eliminate k and m_j from equations (7), (8), (10) and (11) to give

C as a function of β, with e as a parameter:

$$C(\beta; e) = \frac{1}{2\beta}\left[\frac{(1+\gamma\beta)^3(1-e^2)}{(E_0+A\beta)^2} - \frac{1}{k_0}\right]. \qquad (12)$$

Thus $C(\beta; e)$ constitutes a family of curves on the (β- C) plane, each curve being labelled by a value of e. In figure 1 we illustrate this family of curves for the case $\gamma = 0$ (i.e. the mass of the gaseous ring remaining constant) in the main figure and for the case $\gamma = 1$ (i.e. complete merging of the ejected mass with the ring) in the inset. For the case $\gamma = 0$, we have $dm_d/dm_j = 1$. Since we do expect that the dissipated mass carries away some angular momentum, we have illustrated this case in the figure with A = 0.6. In the case $\gamma = 1$, $dm_d/dm_j = 0$, we have illustrated this case in the inset with A = 1. Actually, as will be seen later, the value of A is immaterial to the general conclusion that can be reached from the figure. In both illustrations we have adopted $k_0 = 3$ and $e_0 = 0.25$. Observationally the case of constant mass ($\gamma = 0$) is closer to the reality than the case of complete merging ($\gamma = 1$), at least during the time in which the total emission strength of a line remains constant (e.g. Struve 1931, McLaughlin 1961).

The single-parameter family of curves $C(\beta; e)$ as labelled by parameter e is divided by the critical curve $C(\beta; e_0)$ into two groups. Above the critical curve are curves in the family with $e < e_0$ and below it are curves in the family with $e > e_0$. In the upper region all curves of $e < e_0$ approaches $+\infty$ as $\beta \to 0$ while in the lower region all curves of $e > e_0$ approaches $-\infty$ as $\beta \to 0$. The significance of this family of curves can be seen when we pick a value of C and trace the horizontal line (i.e. C = constant) in the figure. Since the ring starts with $e = e_0$, at $\beta = 0$, the value of e decreases or increases, as β increases at any point, depending upon whether C is above or below $C(\beta, e_0)$. Let us take the case of C = 0. Following C = 0 horizontally

in figure 1, we intercept first curve $C(\beta; 0.2)$ at $\beta = 0.033$, then curve $C(\beta; 0.1)$ at $\beta = 0.077$ and finally curve $C(\beta; 0)$ at $\beta = 0.09$. This shows that as the ejection continues, the eccentricity decreases, until it becomes circular. Once the ring is circular, the level of dissipation as given by C cannot be maintained, because with e = 0 the kinetic energy of the gas in the ring is already completely associated with the angular momentum and cannot be dissipated unless the angular momentum input A assumes a different value. Therefore the region above $C(\beta, 0)$ is forbidden. Physically before the ring becomes circular, the energy dissipation and the angular momentum input will adjust themselves, so it is not realistic to envisage the course of ring modification under the condition of constant C and A at that time. One can easily see that when the eccentricity is close to zero, either the dissipation has to decrease or the angular momentum input has to decrease or both. This means that C and/or A will decrease as the eccentricity approaches zero.

On the other hand if the C is small, say C = -0.2, we find in the similar way as just described that the eccentricity of the ring increases as β increases along the line C = -0.2. But if C lies only a little above the point given by $C(0, e_o)$, we find that the eccentricity of the ring decreases first, then increases, crosses the $C(\beta, e_o)$ curve and finally increases as β increases, because $C(\beta, e_o)$ is not exactly horizontal but is slightly slanted upward with increasing β. But the change in e with β in this instance is very small at any value of β.

From these we can see clearly that for the case $\gamma = 0$ and A = 0.6 that is shown in the figure any $C > -0.1$ tends to decrease the ring eccentricity and any $C < -0.12$ tends to increase the eccentricity. In general large dissipation decreases e and small dissipation increases e.

Let us consider now the effect of the angular momentum input factor A. For samll values of β we derive from equation (12) that:

$$C \to \frac{1}{2}\left[\frac{e_0^2 - e^2}{E_0^2}\frac{1}{\beta} + \frac{1-e^2}{E_0^2}\left(3\gamma - \frac{2A}{E_0}\right) + O(\beta)\right]. \tag{13}$$

Thus for each curve in figure 1 (given e_o, E_o, e, and γ) a change of A and/or γ simply translates it vertically, if we confine ourselves to small β.

For a constant γ, an increase in A corresponds to a translation of all curves downward and a decrease in A a translation upward. Otherwise the relative positions of different curves are not changed. This means that an increase in the angular momentum input has the same effect on the eccentricity of the ring as an increase in the dissipation. Similarly for a given value of A, a change in γ simply translates the entire family of curves upward or downward depending whether γ increases or decreases if β remains small. This means that an increase in γ has the same effect on the eccentricity of the ring as a decrease in the dissipation.

So far we have limited our discussion only to cases of small β, which corresponds to mild ejections that slowly modify the ring structure. What would happen if the ejection is so violent that the ejected mass is much greater than the total mass in the ring? Figure 1 provides us with an answer to this question. If the energy dissipation is high, the eccentricity will be abruptly reduced to zero. But if the energy dissipation is small, a violent ejection abruptly increases e beyond 1 and the ring is completely destroyed.

The variation of k with β can be studied in the same way as that of e. Thus we may express from equations (8), (10) and (11)

$$C(\beta; k) = \frac{1}{2\beta}\left(\frac{1+\gamma\beta}{k} - \frac{1}{k_o}\right), \tag{14}$$

which forms another family of curves on the (β-C) plane with k as the labelling parameter. It can be easily seen that this family of curves is divided into

two groups by a horizontal line

$$C(\beta; k_0) = \frac{\gamma}{2k_0}\ , \tag{15}$$

which is a critical member of the family. All $C(\beta; k)$ curves corresponding to $k > k_0$ lie below this line and all $C(\beta; k)$ curves corresponding to $k < k_0$ lie above that line. All these curves in the family, whether $k > k_0$ or $k < k_0$, are hyperbolas with the two lines

$$\beta = 0 \quad \text{and} \quad C = \frac{\gamma}{2k} \tag{16}$$

as asymptotes. Hence for $k < k_0$, the hyperbolas stack upward as k decreases from k_0 and for $k > k_0$, the hyperbolas stack downward as k increases from k_0. Thus as β increases, k increases indefinitely when $C < \gamma/(2k_0)$, and decreases indefinitely when $C > \gamma/(2k_0)$. In the special case $\gamma = 0$, k increases or decreases according to whether C is negative or positive. Thus we see that the two families of curve $C(\beta; e)$ and $C(\beta; k)$ are very similar. Each family is divided into two groups by a critical curve in its own family, namely $C(\beta; e_0)$ and $C(\beta; k_0)$, e_0 and k_0 being the initial values of e and k of the ring.

By combining the family of curves $C(\beta; e)$, such as in figure 1, with that of curves $C(\beta; k)$, one can obtain at a glance the changes in both k and e at any β for different values of C without actually computing them. In order to do so we may simply draw the two critical curves $C(\beta, e_0)$ and $C(\beta; k_0)$ in the (β-C) diagram. Then the region in the (β-C) diagram above the upper one of these two critical curves corresponds to C and β values that make both k and e decrease as β increases. The region below the lower one of the two critical curves corresponds to C and β values that make both k and e increase as β increases. The region between the two curves includes those C

and β values that will make either k increase and e decrease or k decrease and e increase, depending upon whether $O(\beta\,;\,k_o)$ is above or below $C(\beta, e_o)$. In the case of A = 0.6, $\gamma = 0$, figure 1 shows clearly $C(\beta, k_o)$ which corresponds to the horizontal line, C = 0, is above $C(\beta\,;\,e_o)$. So we have in the intermediate region increasing k and decreasing e as β increases; while in the case of A = 1, $\gamma = 1$, the inset in figure 1 shows that $C(\beta\,;\,k_o)$ which corresponds to the horizontal line $C = 1/(2k_o) = .17$, is below $C(\beta, e_o)$. So the intermediate region on the (β-C) plane includes C and β values that make k decrease and e increase as the star ejects mass.

In this way we have seen how the modification of the ring structure is controlled by three parameters, C, A and γ and found that all combinations of the changes of k and e are possible. Hence we cannot rule out any of the combinations purely from the present consideration.

III. OBSERVATIONAL EVIDENCE OF ORBITAL MODIFICATION

Before we interpret the observational results in the light of ring modification just presented, we may first summarize what has been done with the rotating eccentric ring without orbital modification. First we have developed a quantitative theory of the eccentric ring (of constant e and k) revolving around Be stars with a slow drift of its line of apsides (Huang 1973a). Since it takes from several years to more than a decade for the line of apsides to complete a revolution, the motion of emitting atoms in the ring may be regarded as following Kepler's laws of motion during the exposure time that each spectrogram was taken. Thus from the V/R variation of the emission line and radial velocity curves of the emission edges and the central absorption of the line, we are able to determine the eccentricity and the semi-major axis of the ring. Altogether four stars have been studied in two previous papers (Huang 1973a, ~~and~~ Albert and Huang 1974) from this point of view. From the values $K_{\phi e}$ (in km s^{-1}), which is one half of the separation between red and violet edges of the emission

line, and e, the eccentricity of the ring, determined from observations in those two papers we have recalculated k values (which is denoted by a_1/R in the previous papers) by adopting uniformly 430 km s^{-1} (Slettebak 1966) as the break-up rotational velocity, V, for B3 stars and 562 km s^{-1} as that for B2 stars and 1.8 as the mass-radius ratio (in the solar unit) for B3 stars and 2.0 as that for B2 stars. We have estimated the value of sin i, where i denotes the inclination of the plane of the ring which is assumed to be identical to the plane of the equator, from the observed rotational velocity V sin i and the adopted break-up velocity V. Since according to our interpretation, rings can be formed even when the equatorial rotational velocity is less than V, we may have underestimated sin i, and therefore k. On the other hand according to our theory $2K_e$ should be the difference between the outermost edges of emission lines. But in the actual measurement, the edge is determined by the steepest slope of the density change on the spectrogram, which is way inside the actual edge. Consequently K_e is greatly and systematically underestimated. An underestimate for K_e causes an over-estimate of k.

Table 1 gives the result of this uniform calculation where P_a (in years) is the period of apsidal motion. Only three stars are listed here because the fourth, β^1 Mon, is re-interpreted later in this paper. From the figures in the previous two papers it can be seen that these three stars show good agreement between observations and calculated results for both the V/R variations of the emission line and the velocity curves of the emission edges and central absorption. However in β^1 Mon, we have found that the calculated curves do not agree well with the observed result obtained by Cowley and Gugula (1973). The data of another star, π Aquarii, obtained by McLaughlin (1962), completely defy the interpretation based on the rotating eccentric model of constant k and e. So no attempt was even made by us to understand this star simply on the basis of this model. Indeed the difficulty found in these two stars prompted us to think

that some modification of the rotating eccentric model as has been presented in the previous section is necessary. In light of the modification of the ring we can now proceed to examine the observed results for β^1 Mon and π Aqr.

IV. INTERPRETATION OF β^1 MONOCEROTIS

The major difficulty we have found in the previous investigation of this star is that no ring with fixed values of k and e can satisfy the observation. But if we are prepared to consider the ring as having undergone a few changes in k and e during the period from 1930 to 1970, the eccentric rotating ring can explain the observational data much better than that with constant k and e. The new interpretation is shown in figure 2 which may be compared with figure 3 in the previous paper (Albert and Huang 1974). The improvement is obvious.

In figure 2 where the observed points were taken directly from Cowley and Gugula (1974) we have interpreted the gaseous ring as has gone through three noticeable changes in the time of observation from 1930 to 1970. Three vertical lines show the approximate times of these changes. From the observational data we have found that each change occurs in a relatively short interval so that the calculation (Huang 1973a) based on the eccentric rotating ring with constant k and e is applicable to the period between any two consecutive changes, but we allow for some differences in the values of k and e in different periods. This is equivalent to say that the ejection that is strong enough to modify the ring structure is sporadic rather than continuous. Or more specifically the time interval that the ring structure is being modified by such an ejection is short compared with the period of apsidal motion. However it may or may not be short compared with the orbital period. Previously we have suggested that disappearance of emission lines at certain times in some Be stars is also due to sporadic ejections which destroy the ring completely (Huang 1973b). It goes without saying that ejections that modify the

ring must be mild compared with those that destroy the ring.

In order to present the results of our analysis, we denote by A, B, C, and D the four periods separated by three vertical lines that correspond to the respective times of ring modification. The values of P_a, k and e determined in each epoch are based mainly on the V/R variation. Solid curves representing the V/R variation were first plotted from the determined value of e. The velocity variations are not as clear-cut as the V/R variation, as we can easily see from figure 2. This is understandable if we consider the fact that the value of V/R is determined by the ratio of the number of emitting atoms in the entire approaching portion of the ring to that in the entire receding portion, while the three radial velocities are determined by only small portions of the matter in the ring. Thus any turbulence can greatly distort the radial velocity of particles in the small portion of the ring, but its effect on the V/R variation is somewhat reduced because large portions of the ring are involved. This simply reflects the statistical truth that a small (large) sample corresponds to a large (small) fluctuation.

It appears that observed radial velocities of the central absorption over the entire period of observation can be best fitted to a straight line slightly tilted downward. It means that the radial velocity has decreased a few km s^{-1} in the forty more years. Since this represents, according to the theory,

the radial velocity of the star, such a decrease might represent the orbital motion of the star around the distant companion, if the decrease should be intrinsic. At present we are not sure whether this is so.

Once radial velocity corresponding to the central absorption is determined, the K_e value in each epoch is obtained by finding two horizontal lines placed symmetrically above and below the central-absorption one so that the two fit the observed data respectively of the red and violet emission edges best. The predicted velocity curves for the red and emission edges and central

absorption can then be calculated from K_e and e and are shown as the solid curves in the figure.

In table 2 the values, P_a, k and e thus determined are given for each of these four periods. The eccentricity of the ring in epoch C ~~is~~ can- not ~~well~~ be determined. So ~~[illegible]~~ k is given inside parentheses, uncertain. It appears that from epoch A to epoch B, both the eccentricity and the semi-major axis have increased. But from epoch B to epoch C, the eccentricity ~~is reduced~~ may have undergone a change and finally becomes 0 at epoch D, while the semi-major axis remains practically constant during all these three epochs.

V. π AQUARII

The complexity of the V/R variation and the accompanying radial velocity curves of emission edges and the central absorption of π Aquarii has induced us to propose the ring modification. Indeed once the idea of ring modification is accepted, we have found an easy explanation of the seemingly irregular data obtained by McLaughlin (1962) in the period of half a century. Our interpretation of these data is illustrated in figure 3 where circles represent the observed normal points given by McLaughlin in Table 2 of his paper. Following what has been done in the previous section for β^1 Mon, we have divided the fifty years into thirteen separate epochs denoted by A, B, C, . . . M. This shows that π Aqr is an active star and ejections of matter that are strong enough to modify the ring structure occurred quite often.

In order to determine k from K_e, we must know sin i (where i is the inclination of the plane of the ring) and the mass-radius ratio in the solar unit. According to McLaughlin, π Aqr is a B1 star. So we take the mass radius ratio to be 2.1. In the Appendix we have estimated the observed rotational velocity V sin i to be 480 km s^{-1}. Combined with break-up rota-

tional velocity V equal to 590 km s^{-1} (Slettebak 1966) it gives a rough estimate of sin i = .814. Table 3 gives the values P_a, e, K_e and k determined from the data for different epochs. Observed points of most epochs and the corresponding curves and lines are shown in figure 3. A few epochs (A, H, I, J, K) are not illustrated because there is either no emission or no V/R variations (i.e., zero eccentricity).

VI. DISCUSSION

With the understanding that the ring structure may be modified from time to time, we can now see that stars whose V/R variations have been studied intensively all show this behavior. The only difference is the frequency of these modifications. For stars, like HD 20336 and 25 Ori, the regular periodic variation of V/R and of the radial velocities of emission edges and central absorption can last a few cycles without showing any sign of significant modifications. But π Aqr which shows frequent changes in its emission behavior, is, according to our interpretation, more active in its ejection of matter than other stars studied.

The fact that the data derived from Be stars can all be understood so far in terms of a ring of constant k and e in each interval between two consecutive modifications shows that the disturbances that modify the ring structure last only short times compared with the quiescent periods even for the most active stars observed. That is why we may view the ejections that take place on the surface of Be stars as sporadic. From observations it appears that the time interval between two sporadic ejections is on the average of the order of a few years to a few decades. Whether there are Be stars in which sporadic ejections take place so often as to become practically continuous, we have no idea. If such stars should exist, the V/R variations would become erratic and no analysis of the kind used here can be applied.

The observed data also led us to believe that one and the same star

could be quiescent for a long time and then become active in its ejection of matter later. At present it is not clear whether there is any regular cycle of quiescence and activity for each star.

While the apsidal motion explains fairly well the V/R variation and velocity curves of emission edges and central absorption, we doubt, after the present study, that the apsidal motion is due solely to the equatorial bulge of the star, as it has been thought to be so far (Struve 1931, Johnson 1958). The reason for our doubt arises from the argument that if the apsidal motion should be solely due to the equatorial bulge, its rate of advance would depend critically on the semi-major axis of the ring (e.g. Sterne 1939). As a result the larger the orbit, the less would be the rate of apsidal motion and the longer would be the period P_a. Such a trend has been found neither in ρ^1 Mon nor in π Aqr if we examine tables 2 and 3 carefully. If this trend were not obscured by errors in the determination, we would suggest that the apsidal advance may be influenced by other factors in addition to the gravitational effect of the equatorial bulge of the star.

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APPENDIX

Rotational Velocity of π Aquarii

According to McLaughlin (1962), π Aqr is a B1 star in whose spectrum Mg II λ 4481 "can be measured on plates of the best contrast." But Ringuelet and Machado (1974) have classified the stars as B0 and consequently considered He I λ 4471 as an asymmetrical line instead of a blend of this line with Mg II λ 4481. They have reproduced the tracing of this blend complete with the wavelength scale in their paper in order to show the asymmetry of the line. However when we have followed McLaughlin's interpretation, we can clearly draw from their tracing two rotationally broadened symmetric lines each corresponding roughly to an observed rotational velocity (V sin i) of about 480 km s^{-1} and with a separation that corresponds to that between He I λ 4471 and Mg II λ 4481. Indeed if one should consider the entire blend as one single line, the rotational velocity would amount to more than 700 km s^{-1} which obviously is not reasonable.

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Legends

Fig. 1.--Energy dissipation factor $C(\beta; e)$ as a function of β with e as the labelling parameter. The critical member of this family of curves is $C(\beta; e_o)$ where e_o is the initial value of e when the newly ejected matter starts to modify the ring structure.

Fig. 2.--Interpretation of the observed results (1930-72) of β' Mon on the basis of ring modifications. The points are observed values taken from Cowley and Gugula's (1973) paper while the curves and lines represent our interpretation given in the text.

Fig. 3.--Interpretation of the observed results (1911-61) of π Aqr on the basis of ring modifications. The open circles are observed values taken from McLaughlin's (1962) paper, while the curves and lines are our interpretation given in the text.

Table 1

Characteristics of Gaseous Rings of Three Stars

Star	epoch	P (year)	K_e (km s^{-1})	e	k
105 Tauri	1930-52	11.5	132	.2	3.7
HD 20336	1916-32	4.5	198	.25	4.1
25 Orionis	1914-25	4.8	168	.26	4.7

Table 2

Evolution of the Gaseous Ring Around β^1 Mon

Designation	Epoch	P (year)	K_e (km s^{-1})	e	k
A	1930-54	12.8	200	.3	4.9
B	54-64	11.2	182	.4	6.4
C	64-68	n.d.	171	n.d.	(6.4)
D	68-72	----	165	0	6.5

n.d.: not determinable

Table 3

Evolution of the Gaseous Ring Around π Aqr

Designation	epoch	Pe (years)	K_e (km s^{-1})	e	k
A	1911-23	----	205	0	6.8
B	24-28	5.1	218	.22	6.4
C	29-32	3.3	225	.4	6.8
D	33-36	3.6	241	.35	5.6
E	37-38	no emission			
F	39-41	----	261	0	4.2
G	41-44	3.3	214	.25	6.7
H	44-45	no emission			
I*	47-49	----	351	0	2.3
J	50	no emission			
K	51-54	----	250	0	4.6
L	55-57	4.0	212	.2	6.7
M	58-61	5.5	212	.25	6.8

*This epoch may have witnessed two changes in k. But there are not enough observational points to justify a more refined determination. The eccentricity has remained to be zero during the entire epoch.

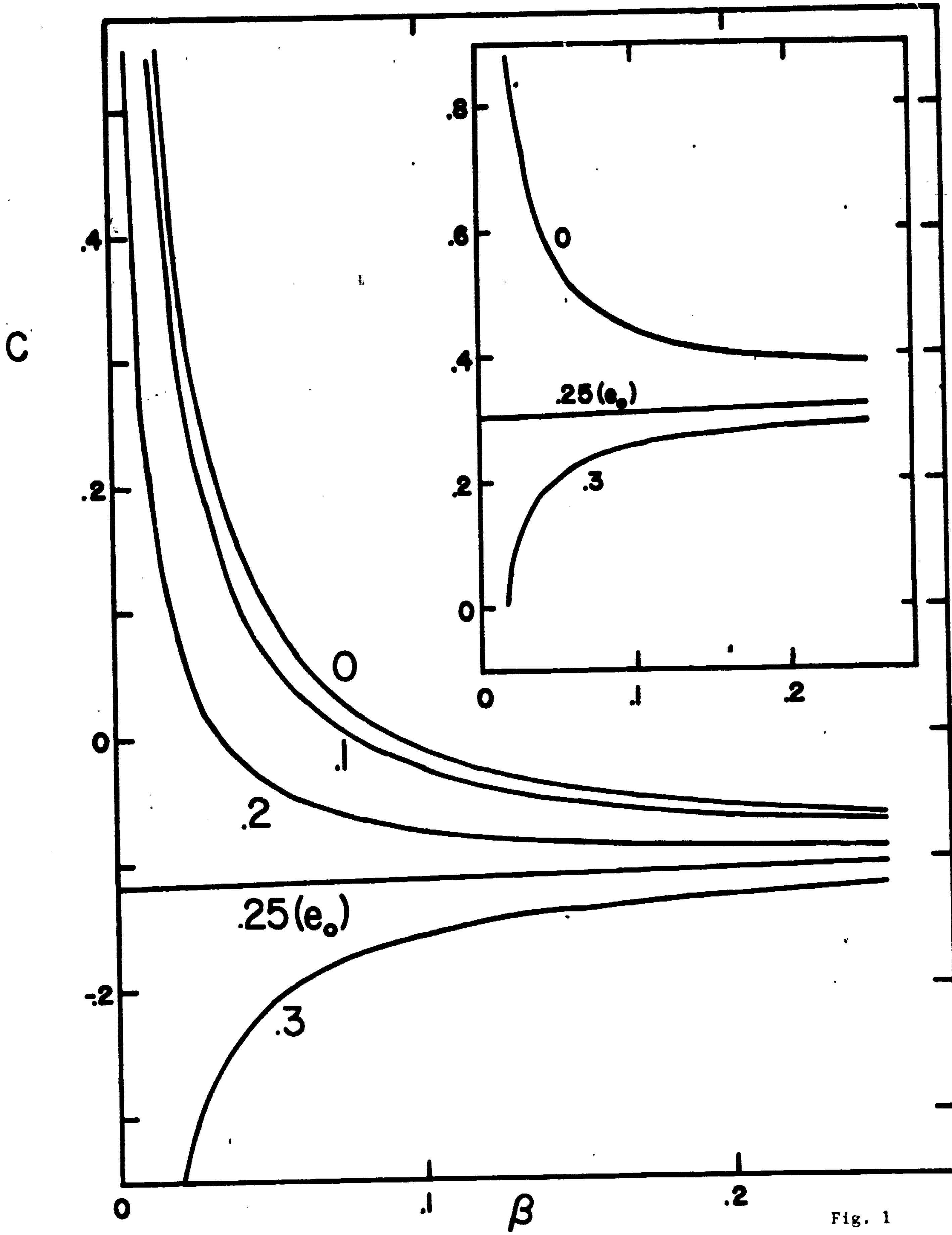

Fig. 1

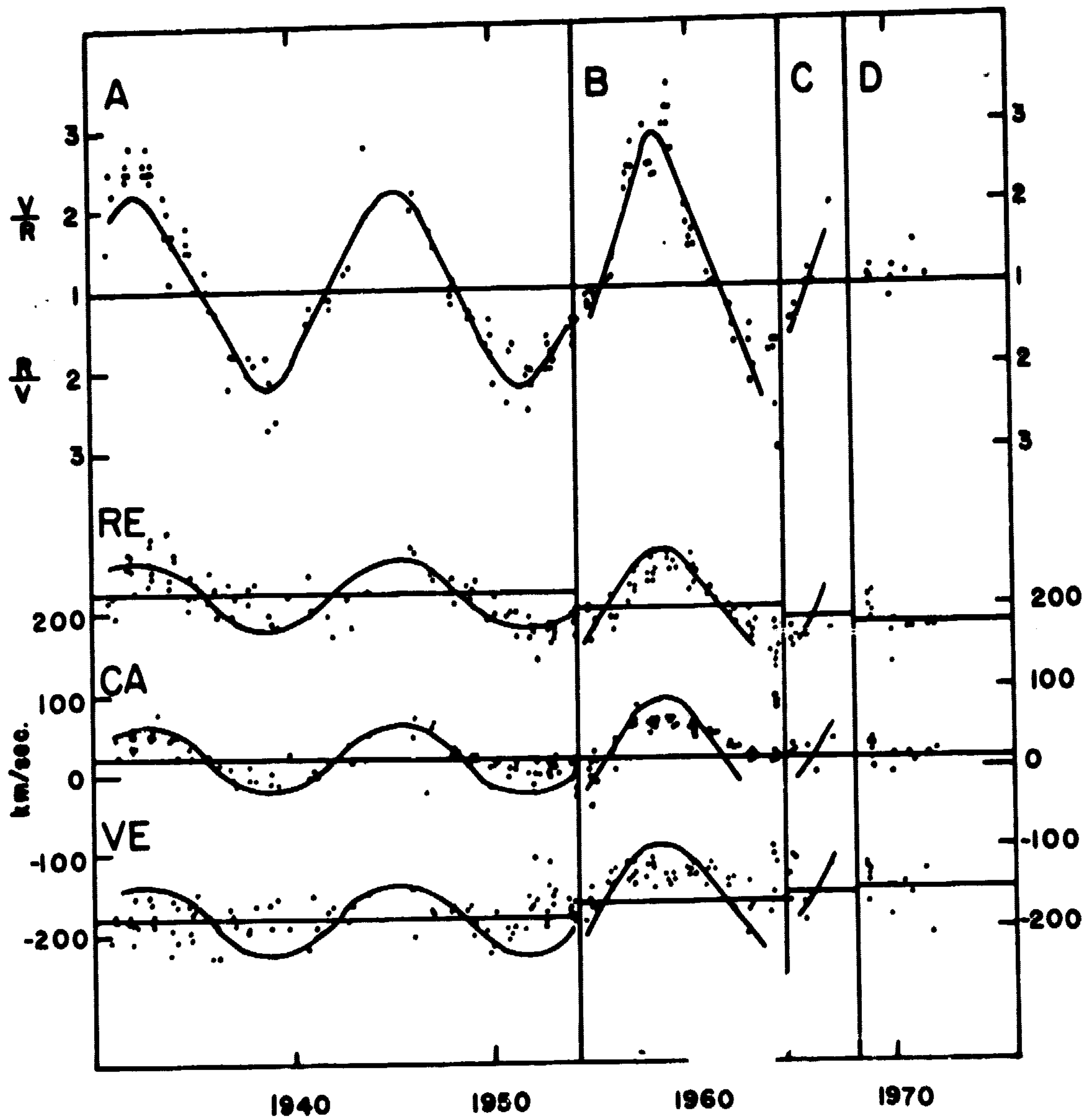
A
B
C
D
V/R
R/V
RE
CA
VE
km/sec.
3
2
1
2
3
200
100
0
-100
-200
1940
1950
1960
1970

Fig. 2

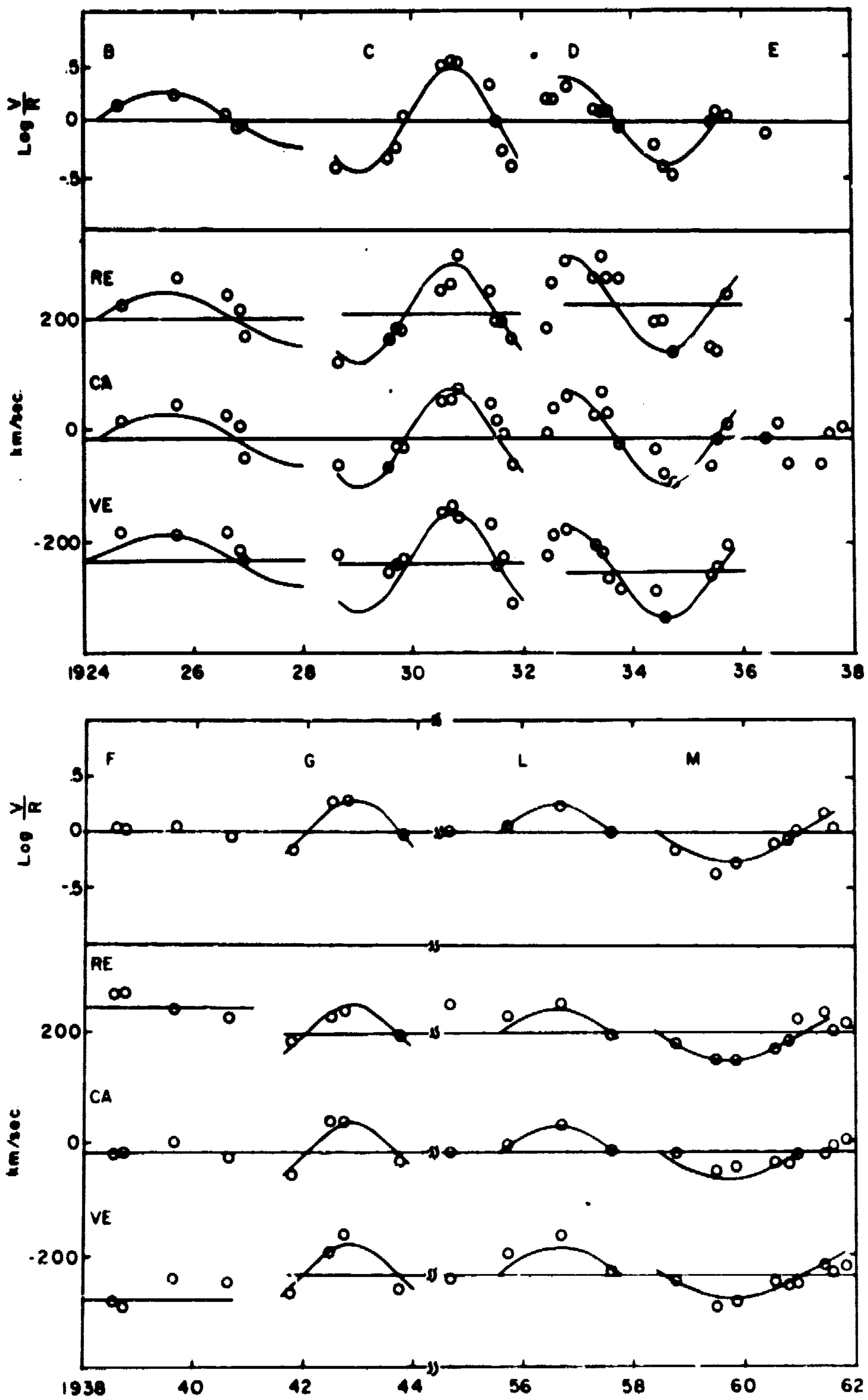
B
C
D
E
Log V/R
.5
0
-.5
RE
CA
VE
200
0
-200
km/sec
1924
26
28
30
32
34
36
38
F
G
L
M
1938
40
42
44
56
58
60
62

Fig. 3